2019 Mock AIME Tiebreaker Round A

by TheUltimate123 and nukelauncher

You are participating in Tiebreaker Round A because you have tied with another contestant and have a score not exceeding 5. The results of this tiebreaker round will determine your placement on the leaderboard.

1 Instructions

1. DO NOT FLIP OVER THIS PAPER UNTIL YOU HAVE STARTED YOUR TIMER.
2. This tiebreaker round is a 5-question, 30-minute examination. All answers are integers ranging from 000 to 999, inclusive. Your score will be the number of correct answers. There is neither partial credit nor penalties for wrong answers. The problems are not necessarily sorted in order of increasing difficulty.
3. Only scratch paper, graph paper, rulers, compasses, protractors, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on this exam will require the use of a calculator.
4. Whenever recording an answer for any problem, record the time, including minutes and seconds, remaining on the clock. PM your most recent answers and times for each problem to both TheUltimate123 and nukelauncher.
5. If multiple scores (number of correct answers) tie, then we will look at the times of the latest correct submission. For instance, consider the following hypothetical scenario:

Problem	Answer	Submission	Time Left
1	031	031	4:13
2	068	065	17:18
3	551	981	1:01
4	137	137	26:24
5	111	151	8:19

Problem	Answer	Submission	Time Left
1	031	031	25:11
2	068	068	19:02
3	551	414	13:13
4	137	123	2:19
5	111	110	0:03

In this case, the two contestants have both solved exactly two problems. However, the latest correct submission for the left contestant occurred at 4:13, while that of the right contestant occured at 19:02. Therefore, the right contestant wins.

2 Problems

A1. Given that $n+2$ and $2n-3$ are the squares of two consecutive integers, what is the sum of all possible values of n?

A2. Suppose that x and y are real numbers such that

$$\log_3(x+y^4) = \log_3(x-y) + \log_3(x+y), \text{ and}$$
$$10 = \log_3(x-2y) + \log_3(x+2y).$$

Find x.

A3. In triangle ABC, $AB = 5$, $BC = 8$, and $CA = 7$. Let the internal angle bisector of $\angle BAC$ intersect $\overline{BC}$ at P, and let Q be the point on $\overline{AB}$ distinct from A such that $CQ = 7$. The square of the area of quadrilateral $ACPQ$ can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

A4. The positive integer N can be written as $\underline{a}\ \underline{b}\ \underline{c}\ \underline{d}$ in base 7, and $\underline{r}\ \underline{s}\ \underline{t}$ in base 9. Suppose that the value of the digit d is the sum of the values of a, b, and c, and that the value of the digit r is the sum of the values of s and t. Furthermore, the values of c and d are double the values of s and t, respectively. Find N.

A5. Aaron and Erin are working together on a 30-question test that consists of 12 physics problems and 18 math problems. Aaron picks two distinct problems at random so that each problem has an equal chance of being chosen, and correctly solves both of them. Erin then chooses one of the 30 problems to solve so that each problem has an equal chance of being chosen. The probability that the problem Erin chose is an unsolved math problem can be expressed in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.