Lynbrook Math Awesome Olympiad

A decade's worth of memes

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May 19, 2021

- The Lynbrook Math Awesome Olympiad (LMAO) is held once or twice every year at Lynbrook Math Club. The problems are written by officers and given to members.
- Contrary to what the name suggests, the LMAO is not an olympiad but rather a short-answer contest.
- Generally, members may compete as individuals or pairs.
- I believe the time limit is usually around 30 minutes, at least in recent years. The number of problems varies.
- Typically, each problem is worth $\frac{1000}{n}$ points, where n is the number of correct submissions to that problem.
- These problems are not heavily edited, so there may be some bugs. They are mostly taken verbatim from their source material, so, for instance, problem 7 from LMAO 2014–15 #2 makes no sense.
- Some problems in 2017–18 were borrowed from other contests such as PUMaC. I'm not aware of this happening in other years, however.
- Answers are provided for the tests for which I could find answer keys.

Contents

§1 LMAO 2010–11

1. On a distant planet, turkeys never die (unless they are killed), and it takes a year for turkeys to grow to adulthood. Every year after turkeys are adults, each pair of turkeys spawns another pair of turkey. One day, two turkeys kill off all the other turkeys in the world the moment they reach adulthood. How long does it take for the turkey population to re-grow to its original size of 400 turkeys?

2. A turkey in a circus is launched out of a cannon at a target composed of four concentric circles of radii 1, 4, 5, and 6. The concentric circles are colored alternately red and white and the innermost circle is white. If the turkey has an equal chance of landing anywhere in the target, what is the probability that the turkey lands on a white region if it hits the target?

3. How many distinct ways are there to rearrange the letters in the word "TRYPTOPHAN"?

4. How many ways are there to place the letters of the word "TOFURKEY" in a 4×2 grid such that no two vowels can be in the same row? (Do not count Y as a vowel.) An example is shown below.
$$\begin{bmatrix} O & T \\ K & R \\ Y & U \\ E & F \end{bmatrix}.$$

5. Felix, Tim, and Tony have decided to run the turkey trot instead of play Starcraft. (The turkey trot is a half-mile race, to be held on a circular track.) When the race starts, Felix begins running at 10 mph, Tim begins running at 9 mph, and Tony begins running at 8 mph. The following complications then occur:
   - Felix drops Draco, his good-luck charm, halfway through the race and does not realize this until he is three-fourths of the way to the finish line. As soon as he finds out, he immediately runs back, picks up Draco, and sprints toward the finish line.
   - Tim's shoelaces become undone when he is halfway through the race, and he pauses to re-tie both shoes. Each shoe takes him ten seconds to re-tie.

   How long does it take each mathlete to run the turkey trot, and who wins?

6. Harry Potter is very ecstatic about a November pizza-eating contest he has decided to enter. The number of pizzas he is able to eat on a given day is the largest prime factor of the date of the month. (For example, he would be able to eat 11 pizzas on the 22nd of the month.) Given that Harry does not know the date of the contest, what is the probability that he will be able to eat at least 7 pizzas?

7. Cynthia is going to eat Xander the Turkey for Thanksgiving. She stuffs his head with her special bread recipe and then cooks him. Before she cooks him, he weighs 100 pounds, of which 95% is actual meat (non-stuffing) by weight. While cooking, however, part of his wing and leg falls off and burns. Luckily, no stuffing was lost. Afterwards, he is 92% meat by weight. How many pounds of tasty goodness will Cynthia miss out on as a result of this tragedy?

8. Xander and Marcus play an ultimate game of Starcraft on Thanksgiving. The two bases are located 60 meters away along a straight course, so that any units running between the bases will fight with units coming the other way. Xander begins to churn out zerglings at

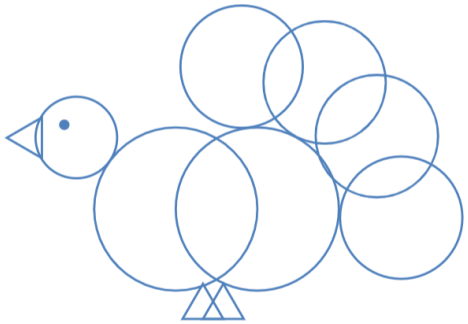
a constant rate of 10 per minute. Each one, after spawning, runs directly towards Marcus' base at 10 m/s. Marcus creates zealots at a constant rate of 6 per minute. Zealots can only run at 5 m/s but are strong enough to kill exactly 2 zerglings. In other words, a zealot will kill the first zergling it sees, with no time lost, but when it reaches the second, they will both immediately die. A player wins when his unit stands in the opponent's base for a full 5 seconds. Who will win when, and after how long?

9. If $f(f(x)) = 2x$ and $f(1) = 3$, what is $\sum_{i=0}^{10} f(2^i)$?

10. Tony the Turkey is jumping on a trampoline. On his first bounce, he reaches a height of 1 foot. On his second bounce, he reaches a height of 2 feet. On his third bounce, he reaches a height of 6 feet. He continues alternately doubling/tripling the height of his bounces (so 1, 2, 6, 12, 36, 72, etc.). If Tim bounces 1000 times, find the sum of the heights of his bounces.

11. There are three piles of candy corn, with each pile containing anywhere from 51-100 pieces of candy corn. No two piles have the same amount of candy corn. Tim the Turkey and Tony the Turkey are playing a game where they can either eat a whole pile, or eat one candy corn from each pile. The turkey who eats the last candy corn wins. Assuming that Tim and Tony always play optimally, and Tim gets to go first, what is the probability that Tim wins?

12. Refer to the diagram of Tim the 2D Turkey. In this picture, all triangles are equilateral and all oval-ish things are circles. The radius of the smallest circle (the head) is $r = 1$. The two large circles (the body) are identical and each have radius 2. In addition, these two circles pass through each other's centers. The medium circles (the tail) have radius 1.5. The lone equilateral triangle (the beak) has side length 1. The two overlapping equilateral triangles (the feet) also have side length one, and the little triangle formed by their intersection is another equilateral triangle with side length $\frac{1}{2}$. The line segments in the tail all have length (??)[1]. Calculate Tim's area.



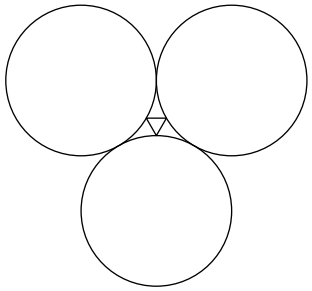
13. Tiebreaker: A prince travels to a distant land and finds three identical princess sisters. The oldest one always tells the truth, the youngest one always lies, and the middle one says whatever she feels like saying. The prince wants to always know what his wife is thinking, so he wants to marry either the oldest or the youngest one. He can ask ONE princess ONE question which has a clear true and false answer. Then, he must choose his wife. Devise a question to help the prince and explain why it works. (Hint: the simpler the question, the better!)

[1] Literally a blank space.

§2 LMAO 2011-12 #1

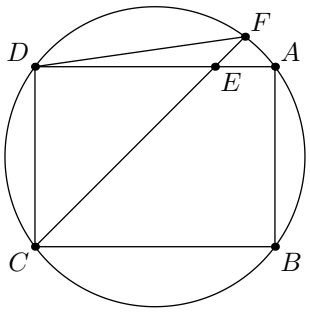
1. What is the area of the largest triangle that can be inscribed in a unit square?

2. The three circles have radius 3 and are externally tangent to each other. What is the area of the equilateral triangle inscribed in the space formed by the circles?



3. There are three bars of white chocolate, five bars of milk chocolate, and seven bars of dark chocolate in a jar. If you can take out a bar at a time and cannot see the inside of the jar, what is the expected number of tries it takes to remove all the white chocolate?

4. E is a point on side AD of rectangle $ABCD$, so that $DE = 12$, while $DA = 16$, and $DC = 12$. If CE extended meets the circumcircle of the rectangle at F, find the measure of chord DF.



5. Julia chased Cynthia around the outskirts of a spherical track of radius r with an nerf gun. The nerf gun can only fire in a straight line with a maximum range of r. What is the probability that Cynthia is within Julia's firing range?

6. Karen and Dennis each thought of a secret integer. Karen's integer is greater than Dennis's integer and their integers satisfy the relations

$$k^3 + d^3 = 35$$
$$k^2 + d^2 = 13.$$

What is Karen's integer?

7. Cynthia is raising an army of miniature people to take over the world. Two-thirds of her army is composed of Shirleys, and the other third is composed of Julias. Half of the Shirleys will be mistaken for Fers, and two-thirds of the Julias will be mistaken for Fers. If a randomly person from Cynthia's army is mistaken for a Fer, what is the probability that it was a Julia?

8. Solve

$$\begin{aligned} x+y+z &= 8 \\ x^2+y^2+z^2 &= 62 \\ \tfrac{1}{x}+\tfrac{1}{y}+\tfrac{1}{z} &= -\tfrac{1}{42}. \end{aligned}$$

9. Find one of the four real solutions to the equation $4x^2 - 40\lfloor x \rfloor + 51 = 0$, if $\lfloor x \rfloor$ is the floor (greatest integer) function of x.

10. Eeyore found his tail in the middle of a circular forest of radius 200 feet. He does not know this, and has no idea how to get out. He is very impatient and will give up after 3 tries, and die a lonely death. On each try, he chooses a random direction and goes for 100 feet, no matter if he is already outside the forest. What is the probability Eeyore will stay lost in the forest forever, at most 100 feet from the center?

11. Tigger is in Paris! Tigger is a pro jumper, and he can jump double the height of his previous jump, starting with a jump of 2 feet, and can go 4 feet in a second. At what time is the sum of all the current heights first greater than the height of the Eiffel Tower of 1063 feet, if he starts jumping continuously at 10:00 am?

12. Winnie the Pooh has found a beehive! He knows that there are many bees, over 9000! (Ten thousand to be exact.) He wants to get the honey, but doesn't know if it's worth the risk. He wants to know if it's worth the risk, and if he has a guaranteed 90% chance of staying alive. The probability that a bee stings Winnie is 0.1%. Write an expression that calculates the probability that Winnie the Pooh will die.

13. BONUS: Move 1 of the following toothpicks so that the expression equals 30.

§3 LMAO 2011–12 #2

1. Find all integers n for which $\sqrt{\frac{4n-2}{n+5}}$ is rational.
2. An isosceles trapezoid $ABCD$ is inscribed in a circle with AC perpendicular to BD. There exists a point O in the triangle such that $OA+OB+OC+OD=12$. What is the maximum area of $ABCD$?
3. If $x^2+y^2=1$ and $x^4+y^4=9/50$, find xy.
4. A diameter CD of a circle is extended through D to external point P. The measure of secant CP is 77. From P, another secant is drawn intersecting the circle first at A, then at B. The measure of secant PB is 33. The diameter of the circle measures 74. Find the measure of the angle formed by the secants.
5. Find the sum of the digits of 101^{12}.
6. Calculate
$$1+\frac{3}{3}+\frac{5}{3^2}+\frac{7}{3^3}+\frac{9}{3^4}+\cdots.$$
7. If the roots of $8x^3+11x^2-3x+5$ are r_1, r_2, and r_3, calculate $\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}$.
8. In the diagram[2] above, $CF=3$, $FG=2$, and $GB=7$. In addition, $[CFA]=9$ and $[BGD]=35$ (where $[XYZ]$ stands for the area of triangle XYZ). Find the ratio $CE:ED$.
9. Dennis is writing down a bunch of real numbers. In his current sequence, every sum of seven consecutive terms is negative, and every sum of eleven consecutive terms is positive. Find the greatest number of numbers that Dennis can write?
10. Karen likes circles, and Karen likes chords. Help her determine the number of ways that, on a circle with 20 points chosen on its edge, she can join pairs of points by nonintersecting chords.
11. Cynthia is cutting a three-dimensional space with planes. If she has 5 planes, how many parts can she cut her space into, at most?
12. Minifer is playing a triangle game with Microfer. On this triangle, ABC (of area 1, for convenience), Minifer chooses a point X on AB. Microfer then chooses a point Y on BC. Then Minifer chooses a point Z on AC. Minifer is attempting to maximize $[XYZ]$. What's the largest area that Minifer can secure?
13. Let a and b be real numbers such that
$$\sin(a)+\sin(b)=\sqrt{2}/2$$
$$\cos(a)+\cos(b)=\sqrt{6}/2.$$
Find $\sin(a+b)$.
14. Dennis's birthday is on Friday. To celebrate, he drew a triangle! What is special about this triangle, though, is that when joining the vertices to the incenter, one of the three triangles formed is similar to the original triangle. What is the largest angle in the triangle he drew?

[2] Do not ask me where the diagram is.

15. A convex hexagon with side lengths 2, 2, 7, 7, 11, 11 (in that order) is inscribed in a circle. What is the diameter of the circle?

16. What is the sum of all integers x for which the triangles with sides $(x, x+2, x+4)$, $(x, x+3, x+6)$, $(x, x+4, x+8)$ are obtuse?

§4 LMAO 2012–13 #1

1. What is 1234×4536271?
2. Johnny Ho currently has 1 gold medal. The other math club officers have none. If Johnny wins another gold medal, how many will they have in total?
3. In their free time, Steven and his friends like to throw around their frisbees. One day, Tony noticed that the diameter of his frisbee was 24 inches, while the radius of Steven's was only 12 inches. Tony proudly proclaimed, "The surface area of my frisbee is n times the surface area of yours!" What was n?
4. Let a and b be two digits such that $\overline{ab}$ is a square. The funny thing is that $\overline{a48b}$ is also a square, and so is $\overline{a4488b}$. What is the square root of $\overline{ab}$?
5. Hexagon $KARINE$ has $KA = IN = 7$, $AR = NE = 23$, $RI = EK = 6$, and $KI = 34$. Given that KR and KN have integer lengths, find the product of those lengths.
6. Jimmery, Ian, and Arkadip are in class 3-B, which has 32 students. Kuroi-sensei calls students to her desk, one at a time, in random order. What is the probability that Ian is the first of the three to be called to Kuroi-sensei's desk?
7. Define a sketchy number to be a number of the form $5^a \cdot 7^b \cdot 11^c$ $(a, b, c \geq 0)$ You have a bag full of sketchy numbers. How many must you draw to guarantee that you can multiply some (at least 1) of them together to get a perfect square?
8. Poor Julia must spend the whole night studying for 7 tests, one in each of her classes: Cooking, Biology, French, APCS, History, English, and Principles of Business. If she insists on studying for Biology after History, (there can be subjects in between) in how many different orders can she study for her tests (and pull an all-nighter)?
9. Find the area of the triangle ABC whose coordinates are: $A = (11, 0, 0)$, $B = (0, 4, 0)$, and $C = (0, 0, 7)$.
10. Solve the equation $(x + 1)(x + 2)(x + 3)(x + 4) = -1$.
11. Daniel has thrown Steven's frisbee up a flight of 14 stairs and Steven must retrieve it. With each step, Steven can either climb 1 stair, or if he's feeling ambitious, 2 stairs. How many different ways can he climb the flight of stairs? (Example: Steven takes 1 step, then 2, then 2, then 1, then 2, then 1, then 2, then 2, then 1.)
12. Let S be the set of points (x, y) such that $x^2 + y^2 - xy \leq 1$. What is the area of S?
13. For how many ordered triples (a, b, c), where a, b, and c are from the set $\{1, 2, 3, 4, \ldots, 2012\}$, satisfy the equation $a^2b + b^2c + c^2a = ab^2 + bc^2 + ca^2$?
14. Keegan has 7 pills. Each day for the next two weeks, he selects a pill from his box. If it's a whole pill, he breaks it in half and takes one half, and puts the other half back into the box. If it's a half pill, he just takes it. Assuming all pills and all half-pills are identical, how many different ways can keegan take his pills? (For example, one way Keegan can take his pills is select all the whole pills the first week, and eat the remaining half pills the next week.)
15. If the function $f(x) = x^3 + ax^2 + bx + c$ has integer coefficients and $\sqrt[3]{7} + 6$ as a root, find $f(1)$.

§5 LMAO 2012–13 #2

1. Joe and Cindy dance together. Joe takes 3 steps every 4 seconds, while Cindy takes 2 steps every 3 seconds. How many steps do they take in all, Joe takes 10 more steps than Cindy?

2. Tony wants to buy some teddy grams for his special someones! He can either choose to buy a bundle of 11 bears for \$3 or a bundle of 15 bears for \$4. How many amounts of bears are impossible to buy? (Example: Tony cannot 8 bears with bundles of 11 and 15.)

3. James and Daisy go out to a restaurant. James and Daisy occupy 1 seat each. Tom arrives, and wants to sit down as well. If $N < 3$, where N is the number of seats at the table, how many unoccupied seats does Tom have to choose from?

4. Johnny is watching a program. He can write 10 lines per second when he is alone, but when he is distracted, he can only write 8 lines per second. If Johnny has to write a 500 line program in 1 minute, for how many seconds does Karen need to distract him? You may assume Johnny does not need to eat or sleep, and that he is never distracted by anyone but Karen.

5. Cars A, B, C, D travel on the same highway, each moving at its own constant speed. A, B, and C are going in one direction and D is in the opposite direction. To begin, A is a distance behind B, who is behind C, and D is far down the highway coming towards them. A passed B at 8 AM and C at 9 AM, and was the first to encounter D, whom he met at 10 AM. D met B at noon ad C at 2 pm. When did B pass C?

6. Emily has 8 love notes: 2 red ones, 2 orange ones, 2 pink ones, and 2 yellow ones. She wants to arrange them in some order to give to Tiny, but she does not want any two consecutive love notes to be the same color. Assuming all love notes are distinct, in how many ways can she do this?

7. It is love week, and Johnny sends flowers to Karen every day. Since Johnny loves math as well, he wants to make sure that the number of flowers he sends each day is greater than the number of flowers he sent the previous day. Additionally, he knows Karen hates prime numbers, so he never wants to send her a prime number of flowers, nor let her have a prime total number of flowers. Flowers cost 1 dollar each, and shipping and handling costs 6 dollars per day. How much money does Johnny need to last 5 days? Of course, Johnny must send a positive number of flowers each day.

8. Arthur has an equilateral triangle ABC, where $AB = 1$. M is the midpoint of AB. Arthur constructs two semicircles, with bases AM and BM, both on the exterior of ABC. Arthur glues this shape onto a non-square rectangular poster, such that his shape shares an axis of symmetry with the poster and touches all of its sides. What is the area of the poster not covered by the shape?

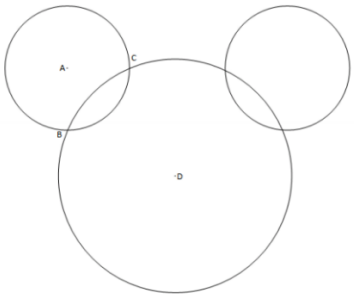
9. Teams A and B play a series of games. Each game has three possible outcomes: A wins, B wins, or they tie. The probability that A wins is $\frac{1}{3}$ and the probability that B wins is $\frac{1}{4}$, and consequently the probability of a tie is $\frac{5}{12}$. The series ends when one team has won two more games than the other, that team being declared the winner of the series. What is the probability that A wins the series?

10. Find the number of pairs (x, y) of real numbers such that

$$16x^2 + 21y^2 - 12xy - 4x - 6y + 1 = 0.$$

11. Keegan is standing at the point $(6, 9)$. There is a wall running along the line $4x + 3y = 12$, whose height is 8. How high does Keegan need to stand to have a good view of Jane and Mary, who are at the origin?

12. For Mickey Mouse, A is the center of the circle of an ear, D is the center of the face, and B and C are the points where circles A and D intersect. If the radius of the ear is 1, $\angle CAB = 90°$, and $\angle CDB = 60°$, what is the area of Mickey Mouse's head?

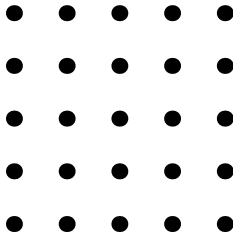


13. What are the last two digits of 3^{44}?

14. What is the sum of the reciprocals of all the factors of 1092?

15. Given that 9^{4000} has 3817 digits and that its first (leftmost) digit is 9, how many positive powers of 9 less than 9^{4000} have 9 as their leftmost digit?

16. Tiebreaker: Try to draw as few circles as possible that pass through all the points in the following 5×5 grid:



§6 LMAO 2013–14

1. I have 3 bananas. If I cut each banana into two pieces, how many pieces will I have?

2. Emily has been kidnapped by Mike, and Tony is trying to rescue her! Tony knows that Mike hid Emily in one of his 5 dungeons, but he doesn't know which one. Tony decides to randomly select a dungeon until he finds Emily. Tony is not very smart, so he might select a dungeon even if he already checked before. How many dungeons should Tony expect to check before finding Emily?

3. Mr. Ramirez is racing Steven in his golf cart. Steven can run at 10 mph, while Mr. Ramirez drives at 100 mph. If Steven had a 45 second headstart, how many seconds does Mr. Ramirez take to catch up?

4. Angela is up all night getting lucky, flipping a fair coin. She wants to flip exactly 3 heads out of her next 5 flips. What is the probability of this?

5. Dr. Rocklin is using a special method to determine which AP Chem test each class period takes. He has 20 different tests, and rolls a fair 20 sided die, as well as a fair 3 sided die. If the test that he gives each class is determined by the number rolled on the 20 sided dice, and the period is determined by the die rolled, and there are 3 chemistry periods, how many different tests can Dr. Rocklin give?

6. Jimmy has adopted a hobby of catching raccoons! He's been spending all afternoon catching racoons. Jimmy decides to take a short nap from 3:00 pm to 4:30 pm. However, at any one given moment in time, some racoon will want to annoy Jimmy, even while he is taking a break from catching racoons. Every 10 minutes, exactly $T/10$ racoons will break into his abode to annoy him, where T is the time elapsed from noon (12:00PM) in minutes. How many racoons will interrupt Jimmy's nap?

7. Victor loves cookies. His cookies are shaped like a regular hexagon with area $6\sqrt{3}$ in^2. He also notices that when he puts his cookie on top of a normal circular cookie, it is as if his hexagonal cookie is perfectly inscribed in the circular cookie. What is the diameter of the circular cookie?

8. Keegan wants to meet a shady person to exchange goods. They will each arrive at their secret meeting location at some random time between 2AM and 3AM, then stay for 10 minutes, then quickly leave into the shadows. What is the probability, in terms of a simplified fraction, that Keegan will meet his shady person to conduct business? (For example, if Keegan arrives at 2:30, his partner can arrive at any time between 2:20 and 2:40 to see him.)

9. Tony has 5 distinct shirts and 5 distinct hats. Tony is a weirdo! He likes to put on shirts, even if he is already wearing a shirt underneath! In fact, Tony is willing to wear up to 5 distinct shirts at the same time. He can also wear any number of hats stacked on top of each other. However, Tony obeys the dress code so he always wears at least one shirt and at least one hat. How many different outfits can he wear?

10. Tony has some marbles and he wants to arrange them in a rectangular grid. If he has 10 marbles, he can arrange them like 1×10, 2×5, 5×2, 10×1. Let $d(N)$ denote the number of ways of place N marbles in a grid. Find the sum $d(1)+d(2)+d(3)+\cdots+d(99)+d(100)$.

11. Find the last 3 digits of 23^{320}.

12. Find the volume of the tetrahedron with vertices $(1, 4, 5)$, $(4, 6, 7)$, $(6, 8, 9)$, and $(5, 5, 7)$.

13. Julia is coming back to visit math club! She starts at the grid point labeled `S` and goes towards the grid point labeled `E`. Because she loves math club and wants to arrive as soon as possible, she goes to math club along the shortest path. However, there may be multiple shortest paths. How many ways can Julia get to math club? (See grid below)

```
S----------#
-#-##-###--#
--------##-#
-##-#-#-##-#
----#---##-#
-##-###-----
-------##-#-
######-----E
```

14. Tony has decided to plant a garden. He has 24 units of dirt, and planting a plant of height N requires N^2 units of dirt. How many ways are there for Tony to plant his plants in a line?

15. Tony is making a cool pattern on the nonnegative lattice points of a grid. He starts by placing 0 at the origin. For each lattice point, he puts the minimum nonnegative number that is not located directly to the left or below of the current lattice point (he fills the lattice points in such a way that the left side and bottom side are always filled first). What number does Tony place at the coordinate $(123, 89)$?

§7 LMAO 2014–15 #1

1. Steven has been given detention by Mr. Ramirez! His punishment is to write on a whiteboard every permutation of the word "RAMIREZ." If it takes him 10 seconds to write a permutation, how many hours will Steven spend in detention?

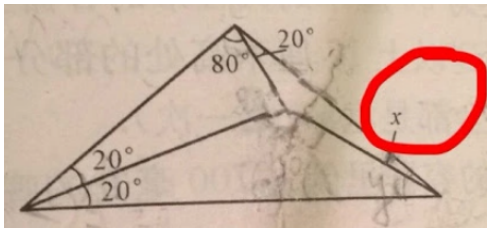
2. Find the area of the triangle with coordinates $B(3, -2)$, $A(-5, -7)$, $E(9, -2/5)$. Put your answer in an improper fraction a/b in simplest form

3. Victor is ordering a burger at Five Guys! He wants a burger and fries, and there are 3 types of burgers, hamburger, cheeseburger, and bacon cheeseburger. There are two types of fries, Five Guys style fries and Cajun style fries. The toppings available for his burger are mayo, Bar-B-Q sauce, lettuce, pickles, tomatoes, jalapeno peppers, grilled onions, A1 steak sauce, grilled mushrooms, ketchup, green peppers, mustard, relish, onions, and Hot sauce. If Victor does not want any peppers or sauce, but can accept any combination of the other toppings (order of toppings doesn't matter), how many burger and fries combinations can he make?

4. Evaluate
$$\frac{1}{1} + \frac{3}{2} + \frac{6}{4} + \frac{10}{8} + \frac{15}{16} + \frac{21}{32} + \cdots$$
where the numerators are triangular numbers and the denominators powers of 2

5. What is the maximum area of a triangle inscribed in an ellipse with semimajor axis of length 4 and semiminor axis of length 2?

6. Find x in degrees.



7. Out of all the integer triplets (a, b, c) such that $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 7/6$, find the unordered triplet which minimizes $a + b + c$.

8. Find all pairs of integers, if possible, of m and n such that $5m^2 - 6mn + 7n^2 = 1985$.

9. Let S be the increasing sequence of positive integers whose base 4 representation has exactly 3 zeros and 5 digits. Find the remainder when the sum of values of S is divided by 1000.

10. On a 16×16 board, Leroy decides to pick n squares so that:
    (i) For each pair of adjacent squares, at least one of them is picked and
    (ii) For every 6 consecutive squares in either a row/column, there are at least two adjacent squares that are picked.

    Find the smallest possible value of n.

§8 LMAO 2014–15 #2

1. If Matthew has 10 boxes and he places an orange and 3 bananas in each box, how many more bananas are there than oranges?

2. Write any real number. You will receive points if your answer is equal to 2 times the most common answer submitted by other people for this question.

3. Given a unit cube, find the number of non-congruent triangles whose vertices are also vertices of the cube.

4. After passing his license test Victor is getting a car! He will only buy a car with license plates that use only the digits 1–9, such that each license plate contains exactly 7 digits. On each plate, all digits are distinct, and for all $k \leq 7$, the kth digit is at least k. Compute the number of valid license plates Victor can get.

5. If it rains today the chance of it raining tomorrow is 70%, otherwise it is 25% of raining tomorrow. If it is not raining today, what is the probability it rains three days from now? Round your answer to the nearest hundredths.

6. William is bored waiting for Purple Comet to start, so he decides to play Victor's piano! If Victor will check his printer every minute, with a 20% chance that the printer finally works so he can print the problems and start the contest 3 minutes later, and William will play 45 notes every minute until the contest begins, what's the expected number keys that William will play? The printer is not working at minute 0.

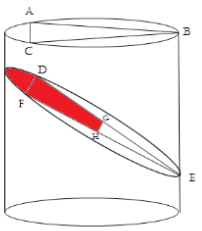
7. Compute the smallest positive integer n such that n, $\lfloor\sqrt{n^2}\rfloor$, $\lfloor\sqrt{n^3}\rfloor$, $\lfloor\sqrt{n^4}\rfloor$, $\lfloor\sqrt{n^5}\rfloor$, $\lfloor\sqrt{n^6}\rfloor$, $\lfloor\sqrt{n^7}\rfloor$, $\lfloor\sqrt{n^8}\rfloor$, and $\lfloor\sqrt{n^9}\rfloor$ are distinct.

8. Matthew and Ajinkya are hungry, and they are facing two stacks of pancakes, of height m and n. Each player, in turn, must eat from the larger stack a nonzero multiple of the number of pancakes in the smaller stack. Clearly, the bottom pancake of each stack is soggy, so the player who first finishes a stack is the loser. For how many pairs (m,n) does Matthew (first player) have a winning strategy if m and n are positive integers not exceeding 10?

9. In triangle PRO, $PR = RO$. A trisector of $\angle R$ intersects PO at M. If PR, PO, and RM are integers and $PR - RM = 7$, compute PO.

10. Find the minimum value of xy, given that $x^2 + y^2 + z^2 = 8$, $xy + xz + yz = 5$, and x, y, z are real numbers.

11. The cylinder above has radius 1. ABC is the inscribed equilateral triangle, and DEF are the projections, respectively, onto a plane that intersects the bases at an angle of 45 degrees. G and H are the midpoints of ED and EF respectively. What is the area of the shaded region?



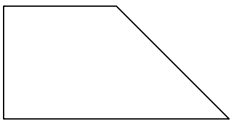
12. BONUS/Tiebreaker: Evaluate the sum of all the digits of A, where A is the sum of all the previous answers.

§9 LMAO 2016–17

1. If 5 moles dig 5 holes in 5 minutes, how many minutes will it take 6 moles to dig 6 holes?

2. A triangle has sides 456, 804, and perimeter 1606. Find the length of the third side.

3. Divide the given region into 4 congruent pieces.



(The given figure is a trapezoid obtained by attaching an isosceles right triangle to a square.)

4. Matthew has two best friends, $\frac{1}{2\sqrt{x}}$ and $-\frac{1}{x^2}$. One day, Matthew realizes that his best friends are opposites of each other! Help Matthew figure out the true value of his friends by computing x.

5. After engaging in passionate _____, Andrew and Aurelia give birth to two Piplups. At least one of them is male. What is the probability that both Piplups are male? (You may also submit a response to what you believe should be in the blank, but you will not be given points for a "correct" response.) (In addition, assume that the probability that a Piplup is male is 1/2, even though it isn't.)

6. How many factors does 792 have?

7. Find the cube root of $15^4 + 28$, given that it is an integer.

8. Andrew is playing Pokemon Go and spots a Snorlax, running 1 km at 18 km/hr to catch it. After catching the Snorlax, he walks back the same distance at a certain speed. If Andrew's average speed was 7.2 km/hr, what was Andrew's walking speed in km/hr?

9. Six people, Victor, Chen, Ajinkya, Nene, Steven, and Hao, are trying to flip water bottles. If Victor has a 1/10 chance to fail his water bottle flip, Chen has a 1/11 chance to fail, ..., Hao has a 1/15 chance to fail, what is the probability they all succeed?

10. Find the smallest positive integer m such that $m^2 + 7m + 89$ is a multiple of 77.

11. William Hu's phone number is `(555)-555-5555`. Find the last three digits of $555^{555^{5555}}$.

12. What is the smallest n such that $999999n = 111\ldots111$?

13. Rectangle $ABCD$ has side lengths $AB = 45$, $BC = 60$. Rectangle $ACEF$ intersects rectangle $ABCD$ in a region with area 1200. What is the perimeter of rectangle $ACEF$?

14. William's Pachirisu, named Tony Ho, has the moves Thunder (which does 110 damage, but only hits 70% of the time) and Rain Dance (which does no damage, but makes it so any future Thunders must hit). If Tony Ho randomly picks a move 2 turns in a row, what is the expected damage done, to the nearest integer?

15. Choose an integer between 1 and 20, inclusive. If your answer is the third highest out of all numbers answered, you gain $\frac{1000}{N}$ points, where N is the number of people who gave the third highest number.

§10 LMAO 2017–18 #1

1. Given that $63^3 = 500^2 + 47$, find 63^6.
2. Austin has the numbers 1, 2, 3, ..., 8 on a whiteboard. Every minute, he picks two numbers at random, takes their product, writes the product on the board, and erases the two original numbers. When there is only one number left on the board, what is the expected value of the number?
3. The five Math Club officers stand in a line for a picture! If Andrew Lin must stand to the left of Stanley Wang (but not necessarily adjacent to him), in how many ways can they stand for the picture?
4. How many ways can one place 6 rooks on a 3×3 board so that there aren't 3 rooks that all attack each other? (Two rooks attack each other if they are in the same row of column, regardless of what is between them.)
5. There are two ordered pairs of positive integers (a, b) and (c, d) that satisfy $x^2+2y^2 = 219$. Find $a + b + c + d$.
6. How many primes p are there such that $p! + 7p$ is a perfect square?
7. King Arthur's 14 bears are having dinner at his round table! There are 7 polar bears and 7 grizzly bears, and they seat themselves randomly around the table for the salmon that is to be served. If King Arthur selects two polar bears at random, what is the probability that they are NOT sitting next to each other?
8. There is a positive integer n such that n^4 is of the form $\overline{1AABAAB}$, where A and B are digits. Find n.
9. Define $\text{BEAR}(p, q) = 2p + 2q + pq$. Find the sum of the first 7 positive integers r such that there exists no pair of positive integers (p, q) where $\text{BEAR}(p, q) = r$.
10. $\angle APB$ is in the plane, such that a circle is tangent to PA and PB at A and B and $PA = PB = 21$. The angle bisector of APB hits the circle for the first time at C. If $PC = 9$, what is the area of the circle?
11. Let r, s, t, u be the roots of the polynomial $x^4 - 3x^3 + 12$. What is $\frac{1}{r^3} + \frac{1}{s^3} + \frac{1}{t^3} + \frac{1}{u^3}$?
12. Triangle ABC has $AB = 58$, $BC = 40$, and $AC = 42$. Point R is the midpoint of AC, and E is the foot of the altitude to AB from C. Find the length ER.
13. In triangle ABC with $AB = 4$, $AC = 5$, and $BC = 6$, let the altitudes from B and C hit AC and AB at D and E respectively. Let the line from A perpendicular to DE intersect the perpendicular bisector of BC at P. Find AP.
14. In triangle ABC with $AB = 13$, $AC = 15$, and $BC = 14$, the midpoint of BC is M and the orthocenter (intersection of altitudes) is H. Let the altitudes from B to AC and C to AB be D and E, respectively. Line HM intersects the circumcircle of ABC at a point P such that A and P are on the same side of BC. Lines AP and DE intersect at G. Compute BG/CG.
15. Every second, Shrek types a random letter with equal probability from the set $S = \{E, G, L, O, R\}$. The expected number of seconds that passes before Shrek sees the string $OGRELOGRE$ can be expressed in the form $p^a + p^b$ for a prime p and integers a and b. Compute $p + a + b$.

§11 LMAO 2017–18 #2

1. What is the sum of all three-digit integers that only use the digits 0, 4, and 8? Leading zeroes are not allowed.

2. Triangle ABC has side lengths of $AB = 10, AC = 14$, and $BC = 16$, and the angle bisectors of the triangle intersect at point I. If area of triangle AIC can be expressed as $m\sqrt{n}$ in simplest form, what is the value of $m + n$?

3. Not long after I travel east on the Dumbarton Bridge on Highway 84, I turn right at the junction of Interstate 880 near Union City. My little son says: "Hey, I realized something - I'd like to ask you this: I am thinking of a right triangle whose legs have lengths 880 and 84 - what is the length of its hypotenuse?" Can you answer my little son's question?

4. How many integers n from 1 to 2018 inclusive satisfy the following property: There exist two not necessarily distinct divisors a, b of n such that $a + b$ also divides n?

5. Compute the sum of $A + B + C + D$, where A, B, C, D are not necessarily distinct digits and $AABB$ and $CDDD$ are four-digit perfect squares.

6. Shrek is on a game show trying to win an autobiography DVD. Initially, there are 5 doors labeled 1, 2, 3, 4, and 5, and the DVD has an $x/15$ chance of being behind door x. Shrek, being bad at math, picks door 1. Fortunately, the game show host now opens doors 2 and 4, revealing that the DVD is not behind either of those doors. If Shrek now has the option of switching, his maximum possible probability of winning is $\frac{m}{n}$, where the fraction is in simplest form. Find $m + n$.

7. Given a triangle ABC with $AB = 8$, $AC = 9$, $BC = 11$, let the circle with diameter BC intersect AC and AB at D, E respectively. Let the tangents to the circumcircle of ADE at D and E intersect at M. Let the second intersection of AM with the circumcircle of ADE be G. If the length of MG can be expressed as $\frac{m}{n}$, find $m + n$.

8. How many integers x, y, z exist such that $-2018 \le x \le 2018$, $-2018 \le y \le 2018$, $-2018 \le z \le 2018$ exist such that $x^3 + y^3 + z^3 = 2018^2$?

9. Define $f(x) = x^2 - 6$. For what range of real number values of x is $f(x)$ closer to 0 than x?

10. If a, b are distinct real numbers such that $ab = \left(\frac{a+b+1}{3}\right)^3$ and $\sqrt[3]{a} - \sqrt[3]{b} = 5$, find $a - b$.

11. **Estimation Question:** Find the smallest positive integer n such that the first digit of 33^{27n} is not equal to 9.

    As a hint, note that

    $$33^{27} = 999715387348960474602494999507529679501 77,$$

    so the answer is clearly not 1!

    Your score for this question will be calculated as follows: If the actual answer be E and your answer is N, your score is $1000 \cdot \min(\frac{N}{E}, \frac{E}{N})^4$.

§12 LMAO 2018–19 #1

1. How many ordered pairs of integers (a, b) satisfy $|a| + |b| \leq 20$?

2. If you connect the points A, B, C, you don't get a triangle with positive area! If the distance $AB = 20$ and $BC = 18$, what are the possible values of the distance CA?

3. Square $DAVI$ has $DA = 4$. Let the circle with center I and radius 5 intersect DA and AV and H and U, respectively. What is the ratio of the area of $HUVID$ to the area of $DAVI$?

4. Math Club is in debt, because not enough people paid for Interstellar. How many ways are there to arrange the letters in STELLAR if the L's refuse to be next to each other?

5. The Math Bowl is a knockout tournament with 32 participating teams. In each round of the Math Bowl, two teams are paired against each other, and the loser is eliminated. If there are 5 questions in each round of the Math Bowl, how many questions are there in total in all rounds of the Math Bowl?

6. Triangle ABC has the roots of the polynomial $2x^3 - 10x^2 + 15x - 6$ as its side lengths. What is the perimeter of ABC?

7. Phoebe needs to study for a test, so she shows up at random time between 1:05 and 1:45 and only stays at Math Club for 5 min. Andrew is tired of the snow in Massachusetts, but he also has to go back very quickly, so he decides to show up at Math Club for only 10 min between 1:05 and 1:45. What is the probability that Andrew and Phoebe will exist at Math Club at the same time?

8. Huvid starts at the point $(0, 0)$ on the coordinate plane and wants to visit Shronkey, who is located at the point $(4, 7)$. However, a line of Farquaad's soldiers is standing on the line $y = x + 3$. If Huvid can only move one unit up or right at each step and cannot cross the line of Farquaad's soldiers (but can stand in the line), how many paths can Huvid take to get to Shronkey?

9. Real numbers x, y, z satisfy $x + y + z = 0$ and $x^2 + y^2 + z^2 = 96$. What is the maximum possible value of $x^3 + y^3 + z^3$?

10. Compute the value of $\sqrt{8 + \sqrt{32 + \sqrt{128 + \sqrt{512 + \cdots}}}}$.

11. **Estimation Question:** Estimate the number of average number of points that all contestants will get from questions 1 to 10. You will receive $\lfloor 1000(0.01)^{\max\{(N/A),(A/N)\}-1} \rfloor$ points, where N is the actual answer and A is the answer you give.

§13 LMAO 2018–19 #2

1. Is 3 a prime number? Is 1.5 a prime number?
2. What is the smallest perfect square containing four distinct nonzero digits?
3. For how many integers n from 1 to 2010, inclusive, is $16n^2 - 19$ divisible by 15?
4. Huvid stacked two unit cubes on top of each other. What is the radius of the smallest sphere that entirely contains both unit cubes?
5. We call a positive integer *basically prime* if it is not divisible by a prime less than 15. I pick all the integers from 1 to n, and take the ratio of the number of basically prime integers to the number of integers that are not basically prime. As n becomes large, to what number does this ratio approach?
6. Given that $\sum_{i=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, compute the value of $\sum_{i=1}^{\infty} \frac{(-1)^n}{n^2}$.
7. Let circle C have center $O(0,0)$ and radius 12. Let P and Q be the intersections of C and $y = 2x$, and let R and S be the intersections of C and $y = \frac{1}{3}x$. What is the area of the convex polygon with vertices P, Q, R, and S?
8. Austin and Stanley are standing on two opposite corners of a square. At each second, both of them will either stay put or move to one of the two adjacent vertices, with all possible moves having probability $\frac{1}{3}$. If they can travel through each other, what is the probability that they will be on the same vertex in 2 seconds?
9. Let $P(x) = (x+1)(x-1)(x-2)(x-3)+4$. What is the sum of the fourth powers of the roots of $P(x)$?
10. In triangle ABC with circumcenter O, let P be the second intersection of lines AO and the circumcircle of triangle BOC and let J be the incenter of PBC. If $AB = 13$, $BC = 21$, $CA = 20$, compute the length AJ.
11. **Estimation Question:** Call a number *cool* if the number formed by its first 1 digits is divisible by 1, the first two digits is divisible by 2, ..., the first n digits is divisible by n, where n is the length of the number. For example, 12365 is cool since 1 is divisible by 1, 12 is divisible by 2, 123 is divisible by 3, 1236 is divisible by 4, and 12365 is divisible by 5. How many digits does the longest cool number have? If your answer is X and the correct answer was Y, your score will be $\max\left(0, \frac{200(5-|X-Y|)}{\max(3,Z)}\right)$, where Z is the number of teams/individuals that get a nonzero score on this problem.

§14 LMAO 2019–20

1. For how many positive integers n does $1 + 2 + 3 + \cdots + n$ evenly divide $5n$?

2. Four consecutive positive integers have the property that the sum of the cubes of the smaller three integers is equal to the cube of the largest integer. What is the value of the sum of these four integers?

3. Jovial Jonathan would take ten hours to grade a stack of CAML's alone, and Dauntless David would take 12 hours to grade them alone. When they work together, they talk a lot, and their combined output decreases by 10 CAML's per hour. Working together, they grade all CAMLs in 6 hours. How many CAMLs are in the stack?

4. Math Club has a stack of 10 real one-dollar bills and 10 counterfeit bills. Math Club will deposit 10 of the bills into the bank. Due to insider information, Math Club knows that the banker is lazy and will only randomly select a one-dollar bill to inspect. If the bill is counterfeit, the banker has a 50% chance of finding out and nullifying Math Club's deposit. Otherwise, Math Club will gain money equal to the number of counterfeit bills deposited. How many counterfeit bills should Math Club use to maximize the expected amount of money gained?

5. How many of JonQ's following statements are true?

   I. Statement V is false.

   II. More than half of these statements are true.

   III. This statement is true unless statement II is true.

   IV. All the prime number statements are false.

   V. Between statements II and VI, exactly one is true.

   VI. When JonQ says that this statement is false, he is lying.

6. Isosceles triangle ABC with $A(1, 1)$ and $B(-3, 9)$ has all vertices lying on the parabola $y = x^2$. How many distinct possible values are there for the coordinates of point C?

7. All convex polygons with perimeter 1 have area less than x. What is the smallest possible value of x?

8. Jeremy is waiting for his pizza, which will arrive uniformly at a random time between 1pm and 2pm. He's lazy, though, and will only wait for a random 10-minute time interval within that hour. The pizza delivery driver will wait for 15 minutes after arriving before leaving. What is the probability that Jeremy will get his pizza?

9. Find the value of
$$\frac{1}{3} + \frac{1}{3^2} + \frac{2}{3^3} + \frac{3}{3^4} + \frac{5}{3^5} + \frac{8}{3^6} + \cdots.$$

10. What is the maximum possible area of a semicircle inscribed in a square of side length 1? Express your answer in the form $(a - b\sqrt{c})\,\pi$, where a, b, c are positive integers and c is square-free.

11. **Estimation Question:** What is the smallest positive integer value of x such that $1000! < x^x$? If the correct answer is x and you submit positive integer y, your answer will be $\max\left(0, 100 - 800\frac{|y - x|}{x}\right)$ % of the maximum possible score. Otherwise, your score is 0.

§15 LMAO 2020–21

1. The distance between San Jose and Sacramento is 121 miles by car or 89 miles by air. David the driver will race Bob the bird from San Jose to Sacramento. What percent higher must David's average velocity be than Bob's in order to win the race? Round your answer to the nearest integer.

2. After giving a series of particularly engaging presentations, Kevin is being scouted by corporate sponsors! In particular, if Kevin gives x presentations, company A is offering him $2x^2 - 8x + 6$ dollars, and company B is offering him $12x + 2$ dollars. If Kevin can only accept one of the two companies' deals, and is planning on giving between 5 and 10 presentations (inclusive), what is the maximum amount of money he can make? Express your answer in dollars.

3. Consider two points $A = (x_1, y_1)$ and $B = (x_2, y_2)$ in the coordinate plane, where x_1, x_2, y_1, and y_2 are chosen uniformly at random from the interval $[0, 1]$. Let $p = \frac{m}{n}$ denote the probability that the x-axis, y-axis, and line AB enclose a triangular region contained entirely in the first quadrant, where m and n share no common factors. What is $m + n$?

4. Sumedh's orchard is 20 miles east and 20 miles north of where James lives. To get there as quickly as possible, James decides to ride his father's teleporting car. This car, however, can only travel half the distance east as it does north (for example, 5 miles east and 10 miles north), ostensibly to ensure that James gets his proper exercise by walking the remaining distance. Let d be the minimum distance (in miles) that James still needs to walk to get to his destination. Find d^2.

5. There are six distinct ducks in a pond, and each duck quacks at exactly one of the other five ducks. Across all possible configurations of quacks, let the average number of triples of ducks A, B, C such that duck A quacks at duck B, duck B quacks at duck C, and duck C quacks at duck A be $\frac{m}{n}$ in simplest form. (Two triples are considered different if there is a duck on one of them but not the other.) What is $m + n$?

6. Kevin is challenging a friend to a chess duel. Kevin wins the duel overall if he wins two matches in a row before losing three matches in a row. Since his friend is imaginary, both have a 50% chance of winning each match. Given that Kevin loses the first match, let the probability he still wins the duel be $\frac{m}{n}$ in simplest form. What is $m + n$? Assume contestants have infinite stamina.

7. Let $ABCD$ be a parallelogram. The circle with diameter BD intersects AB and AD at points P and Q respectively. If $AC = 60$, $BD = 24$, and the lengths of the four segments AP, AQ, AB, and AD are four distinct integers, what is the perimeter of $ABCD$?

8. I have three potions each containing salt, pepper and garlic. Potion High has salt : pepper : garlic in the ratio $2 : 1 : 1$, Portion Medium in the ratio $1 : 2 : 1$, and Potion Low in the ratio $1 : 1 : 2$. I choose three positive numbers x, y, z with $x + y + z = 1$ uniformly at random. Then the probability that, by mixing appropriate amounts of each potion, I can create a new mixture with ingredients in the ratio salt : pepper : garlic $= x : y : z$ is $\frac{m}{n}$ in simplest form. What is $m + n$?

9. (Estimation) On Facebook Messenger, there are 1719 emojis. Each day, I pick a random one. What is the expected number of days it takes me to finish picking every emoji at least once?

§A Answers

§A.1 LMAO 2011–12 #1

1. $1/2$
2. $63\sqrt{3}/4 - 27$
3.
4. $10\sqrt{2}$
5. $1/3$
6. 3
7. $2/5$
8. $(-2, 3, 7)$
9. $\sqrt{29}/2$, $\sqrt{189}/2$, $\sqrt{229}/2$, $\sqrt{269}/2$
10. $1/4$
11. 10:04:26 am
12. $1 - 0.999^{10000}$
13. $141 - 111$

§A.2 LMAO 2011–12 #2

1. 13
2. 12
3. $\pm\sqrt{41}/10$
4. 60
5. 82
6. 3
7. $-101/25$
8. $9/16$
9. 16
10. 16796
11. 26
12. $1/4$
13. $\sqrt{3}/2$
14. $720/7$
15. 7
16. 5

§A.3 LMAO 2012–13 #1

1. 5597758414
2. 2
3. 1
4. 7
5. 812
6. $1/3$
7. 4
8. 2520
9. $93/2$
10. $x = (-5 \pm \sqrt{5})/2$
11. 610
12. $2\pi\sqrt{3}/3$
13. 12140408
14. 429
15. -133

§A.4 LMAO 2013–14

1. 6
2. 5
3. 5
4. $5/16$
5. 6840
6. 225
7. 4
8. $11/36$
9. 105625
10. 482
11. 201
12. $1/3$
13. 18
14. 2024
15. 34

§A.5 LMAO 2014–15 #1

1. 7 hours
2. $43/5$
3. 6144
4. 8
5. $6\sqrt{3}$
6. 10
7. $(1, 5, -30)$
8. none
9. 962
10. 153

§A.6 LMAO 2014–15 #2

1. 0
2. 20
3. 3
4. 360
5. 0.41
6. 2187
7. 146
8. 29
9. $1/2$
10. 16384
11. $\pi\sqrt{2}/3 + 5\sqrt{6}/16$

§A.7 LMAO 2017–18 #1

1. 62523502209
2. 40320
3. 60
4. 6
5. 36
6. 1
7. $11/13$
8. 34
9. 32
10. 400π
11. $3/4$
12. 21
13. $8\sqrt{7}/7$
14. $5/9$
15. 18

§A.8 LMAO 2017–18 #2

1. 11592
2. 17
3. 884
4. 1009
5. 16
6. 19
7. 147
8. 0
9. $(-3, 2) \cup (2, 3)$
10. 35
11. 371

§A.9 LMAO 2018–19 #1

1. 841
2. 2, 38
3. $31/32$
4. 1800
5. 155
6. 5
7. $43/128$
8. 165
9. 384
10. $1 + 2\sqrt{2}$[3]

§A.10 LMAO 2018–19 #2

1. Yes, No
2. 1296
3. 536
4. $\sqrt{6}/2$
5. $192/809$
6. $-\pi^2/12$

[3]See https://aops.com/community/c436035h1744324_tragic_mistake for a fun story.

7. $144\sqrt{2}$

8. 20/81

9. 83

10. 65/3

11. 25

§A.11 LMAO 2019–20

1. 3

2. 18

3. 600

4. 10

5. 2 or 3[4]

6. 6

7. $1/4\pi$

8. 91/240

9. 3125

10. 3/5

11. $(3-2\sqrt{2})\pi$

12. 874

§A.12 LMAO 2020–21

1. 36

2. 126

3. 3

4. 80

5. 33

6. 8

7. 128

8. 17

9. 13798.42075

[4]The problem was incorrect and there was no correct answer.